OPTIMIZATION OF A MULTIMODE SERVO-MECHANISM WITH ADJUSTABLE INITIAL CONDITIONS

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THESIS

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ABSTRACT

A study of a multimode third order instrument servomechanism is presented. Particular emphasis is placed on the effects of the initial condition of a compensating network, switched in as the system enters a linear operating mode, upon the behaviour of the output. The optimum values for the initial condition as a function of velocity and acceleration of the servomotor is derived, using the integral squared error as a criteria. The results are confirmed with a computer simulation.

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List of Symbols and Abbreviations

ζ	=	Damping factor of a complex pole second order system
ω_n	=	Natural frequency of the system
τ	=	Time constant of a system, viz.: = RC
K_v	=	Bode gain of type one system, or velocity constant
$\dot{x}(t)$	=	$\frac{dx(t)}{dt}$
$\ddot{x}(t)$	=	$\frac{d^2x(t)}{dt^2}$

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I. INTRODUCTION

Multimode control systems are those which are operated under two or more controlling laws which are in turn switched on or off according to some pre-established rule. Here, a particular case is studied, in which an instrument-servo is used to position a device in two main steps: 1.- First the motor is driven open loop at full forward speed, and then slowed down by reversing its voltage — both without feedback; 2.- Finally, within a given error limit, a feedback loop is closed, zeroing the error according to the linear-system response.

It is the second of these steps that is studied with greater detail as it is here where the optimization technique of this work applies.

In general, in this type of system a third order type one instrument servo is used which is unstable under the desired gain conditions. Therefore, a compensating network has to be added to the forward path of the loop which is a lead network in this case. In practice, the initial condition on the capacitor of this lead network is usually made equal to zero, as it is easier to have a short circuit across the capacitor than to provide an extra voltage source for this initial condition. No investigation has been found in the literature as to the effect of this initial condition on the time response of this type of control, and it is the purpose of this study to determine its effect on the servo performance.

To accomplish this, a typical third order instrument servo of the linear actuator type has been modeled and the whole problem simulated with the aid of the System/360 Continuous System Modeling Program of IBM (S/360 CSMP) in the IBM 360/67 Computer.

The results obtained from this special case simulation will be discussed in the light of producing a more general criteria for similar cases.

The performance criteria to be used will be that of the Integral Squared Error, since this function provides the necessary information on minimum error which is desired here. Minimum settling time will be observed from the data obtained from the computer simulation.

II. BASIC CONCEPTS ON DISCONTINUOUS SYSTEMS

Consider first a general block diagram of a discontinuous control system as shown on Fig. 2.1. The general control is achieved by the control computer which can be an analog or digital computer, an amplifier with relay, or, a simple relay circuit. The controlling signals may come from any point in the system, the input and output being the most frequently used. The control computer uses its input signals to provide the desired outputs which control the parameters of the plant, the controller and amplifier, or the feedback signals.

From the analysis of the input signals, the input signal analyzer provides the required signals for the control computer. This analyzer may consist of a differentiator or integrator network, a sampler, a potentiometer or a function generator.

The controller and amplifier network may consist of an amplifier with relay with R-C network, or a low power amplifier circuit which provides a compensated signal for the plant drive. The control computer can control the gain of the amplifier and the compensating circuit.

The plant is the object to be controlled by the rest of the devices. In electrical control systems it may consist of a motor driving a load, its shaft being designed to follow the input signal. The plant must also be designed to meet the specifications of the system, which in the given case would be settling time, overshoot, transient response, etc. The control computer can vary the gain, the damping and/or the inertia of this plant.

The feedback block consists of one or many signals which are a function of the outputs. The design will consider the number of signals

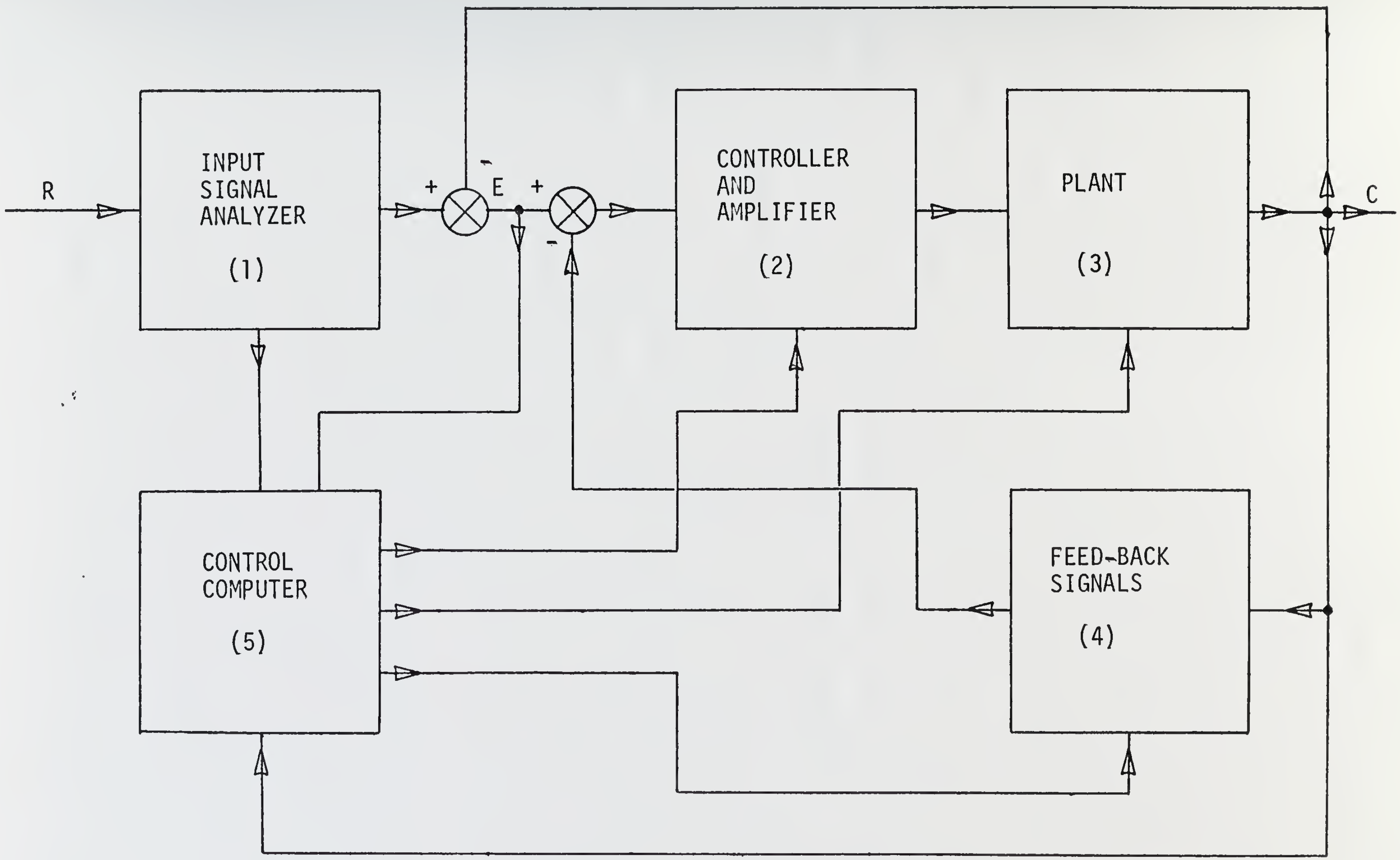

Fig. 2.1. General Block Diagram For a Discontinuous System

to be used, and the control computer can control the polarity, instant and period for them to be applied, as well as their gains and character.

From this general structure, many special cases can be derived by placing limitations or specifications on the blocks indicated above.

By using the controller and amplifier block and the plant, various control methods have been developed. The basic theory along these lines is to design the controller to make the switch operate along a curve in the phase plane, or a surface in the phase space that is called the optimum switching curve or hypersurface (Ref. [1]).

For the continuous operating mode of the system, a compensating network is used, and if blocks 2 and 3 are used, the compensation is the cascaded type. This form of compensation seems predominant in this field, although other types of compensation could be used. The theory of cascaded lead - lag networks has been largely discussed and developed, the Bode diagram being used with great advantages in the design.

The reason for using discontinuous systems arises from the need of applying a limited amount of power to accomplish traveling in minimum time. These types of systems have been proven to be the best form of control even under economic considerations.

Discontinuous systems are based on the theory that it is best to use maximum drive at the beginning of the trajectory, and then brake the system at a proper point. Thus, full voltage is applied at the beginning, and full reverse at the switching point. With the discontinuity, the character of the system is entirely different when the state point is at different sides of the hypersurface. The decaying part of the trajectory should be made as close to the E vs $\dot{E}$ plane as possible Ref. [2].

Comparing the continuous system with the discontinuous, the phase plane plot of E vs $\dot{E}$ serves best to give a better insight of the difference. Fig. 2.2 shows a typical phase plane of a discontinuous system, whilst Fig. 2.3 presents that of a second order continuous system.

The continuous system is normally designed to have some overshoot in the trajectory, because an operation in the overdamped or even critical condition provides a slow response. In the discontinuous system the acceleration trajectory can be made unstable, and the die out trajectory a straight line near the $\dot{E}$ axis in the phase plane (fast eigenvector).

This general discussion has only concerned itself with possible ways to use discontinuous theory to improve the response character of systems. The general switching theory and response character will not be discussed here in more detail. The main concern in later chapters will be the effect of entering a final phase which will be continuous; the use of a lead type conpensating network, and the influence of the initial conditions, in particular that of the compensating network.

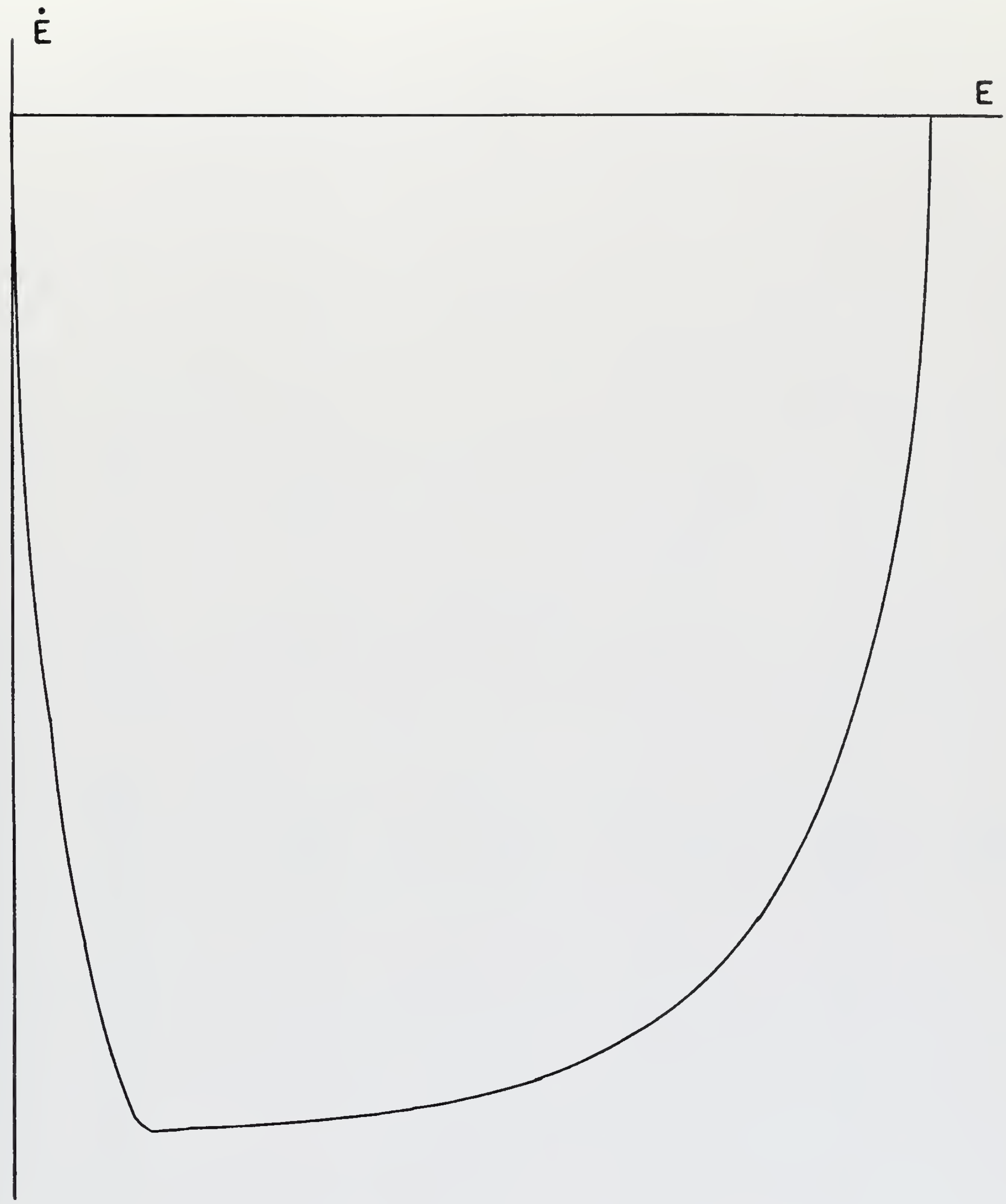

Fig. 2.2 Typical E vs $\dot{E}$ Plot for a Discontinuous Operating System

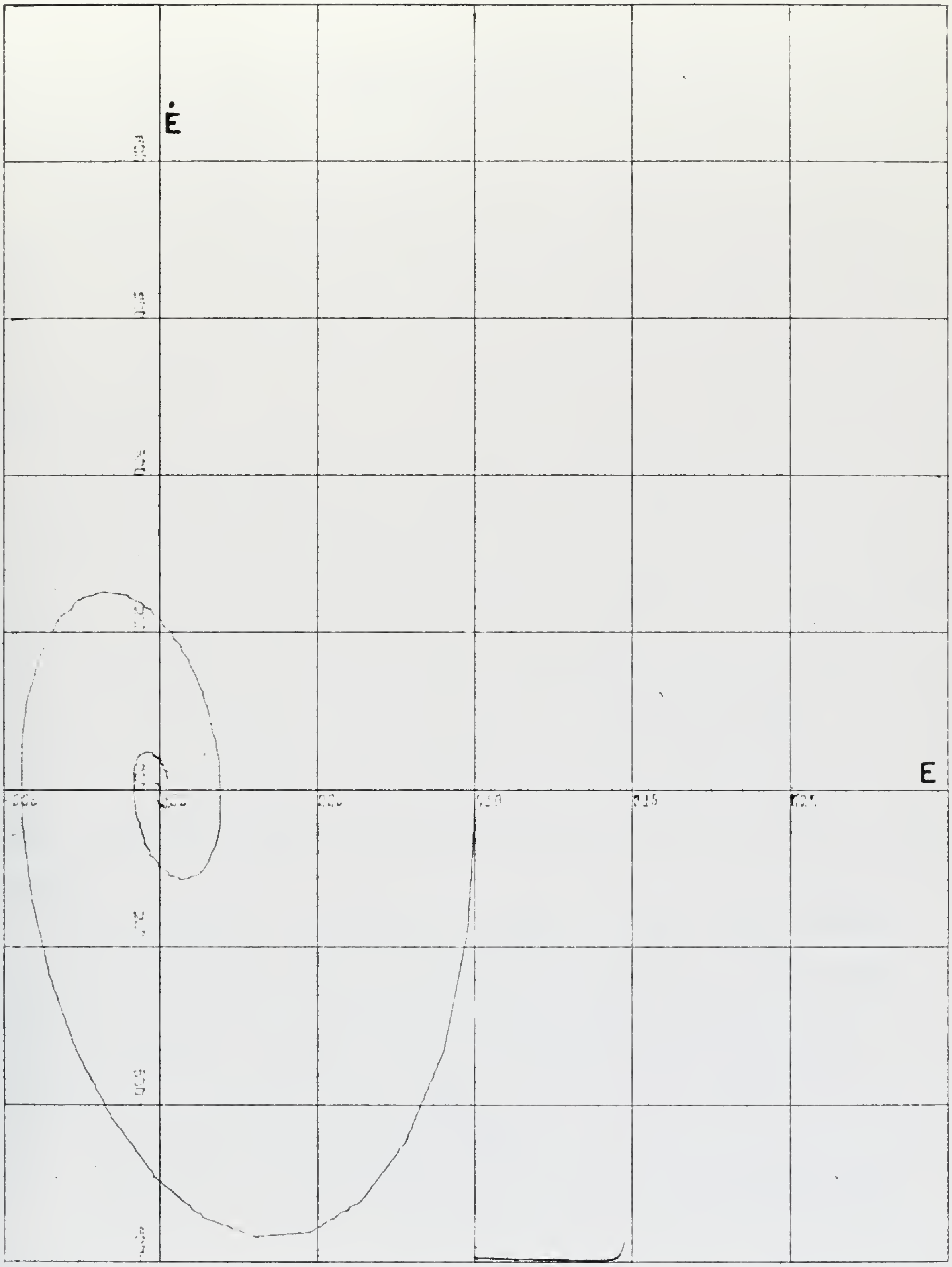

Fig. 2.3 Typical E vs Ė Plot for a Second Order System

III. PLANNING THE EXPERIMENT

A. ANALYTIC DISCUSSION

In order to study a case of how the initial conditions on the lead network would affect the performance of a system, the problem was simulated using the characteristics of a linear actuator, such as the ones used in disc memories for computers to move the arm which carries the magnetic head.

The system is supposed to move different distances, the only known variables being the initial error and the actual position of the head at all times. Minimum time and final error are of the essence, thus leading to a discontinuous bang-bang type of operation. The final part of the trajectory, i.e.: when the head reaches the edge of the track to which it was ordered, is accomplished in a closed loop mode. An extra gain is introduced to the system, and thus a filter has to be added in cascade in order to achieve stability, the filter being a lead network.

Once a switching is determined, in order to have a real starting condition of velocity and acceleration of the motor as it enters the closed loop mode, a computer simulation using the S-CSMP/360 was implemented to study this final phase.

1. Equations of the Physical System

The known characteristics of the motor in practical MKS units were:

Force constant = $K_m = 15.6 \dfrac{\text{Newton}}{\text{Ampere}}$

Motor inductance = L_a = .01 [Henry]

Motor resistance = R_a = 5 [Ohms]

Mass of motor = M = 0.45 [Kg.]

Viscous damping = f = 17.5 $\frac{\text{Newton}\cdot\text{sec}}{\text{Meter}}$

Back EMF constant = K_b = 15.6 $\frac{\text{Volts}\cdot\text{sec}}{\text{Meter}}$

Armature voltage = v_a [Volts]

$$M\ddot{x} + f\dot{x} = K_m i_a \tag{3.1}$$

$$K_b\dot{x} + L_a\dot{i}_a + R_a i_a = v_a \tag{3.2}$$

which, using Laplace transforms and substituting for i_a, gives

$$V(s) = \left[(s^2M + sf)\frac{R_a}{K_m} + \frac{sL_a}{K_m}(s^2M + sf) + sK_b \right] X(s)$$

or

$$G(s) = \frac{X(s)}{V(s)} = \frac{1}{s\left[s^2\frac{L_aM}{K_m} + s\left(\frac{MR_a}{K_m} + \frac{L_af}{K_m}\right) + \frac{fR_a}{K_m} + K_b \right]} \tag{3.3}$$

which is the transfer function for the motor with voltage source feed.

Similarly, the study could be made by considering a current source feed, in which case the transfer function would be

$$\frac{X(s)}{I(s)} = \frac{K_m}{s(sM + f)} \tag{3.4}$$

Going back to the voltage feed transfer function, and substituting the given values of the motor characteristics, the open loop

gain is:

$$G(s) = \frac{3432.3}{s[s^2 + 538.6s + 72741]} \tag{3.5}$$

From this expression, and by simple inspection, it is known that

$$\omega_n = 269.7 \frac{rad}{sec}$$

$$\zeta = 0.9985$$

for the quadratic factor in the denominator.

Thus the servomechanism is a type one, third order system, with two complex poles almost critically damped. The Bode diagram for such a motor is shown in Fig. 3.1.

From the Bode diagram, it is seen that the K_v of the motor is of the order of 0.047, which will represent an exceedingly slow system for the final closed loop phase of the operation, as compared with the desired settling time.

2. Filter Design

By further inspection of the Bode diagram, it is determined that an amplifier which should provide an extra gain of 55,000 ought to be used. This new characteristic function gives an unstable closed loop system, which in turn leads to the necessity of adding the lead network.

A general and most simple form of lead network is represented by

$$G_c(s) = \frac{\alpha(s\tau + 1)}{\alpha s\tau + 1} \tag{3.6}$$

and its physical implementation is shown in Fig. 3.2, where α is the relationship between the corner frequencies of the filter as referred

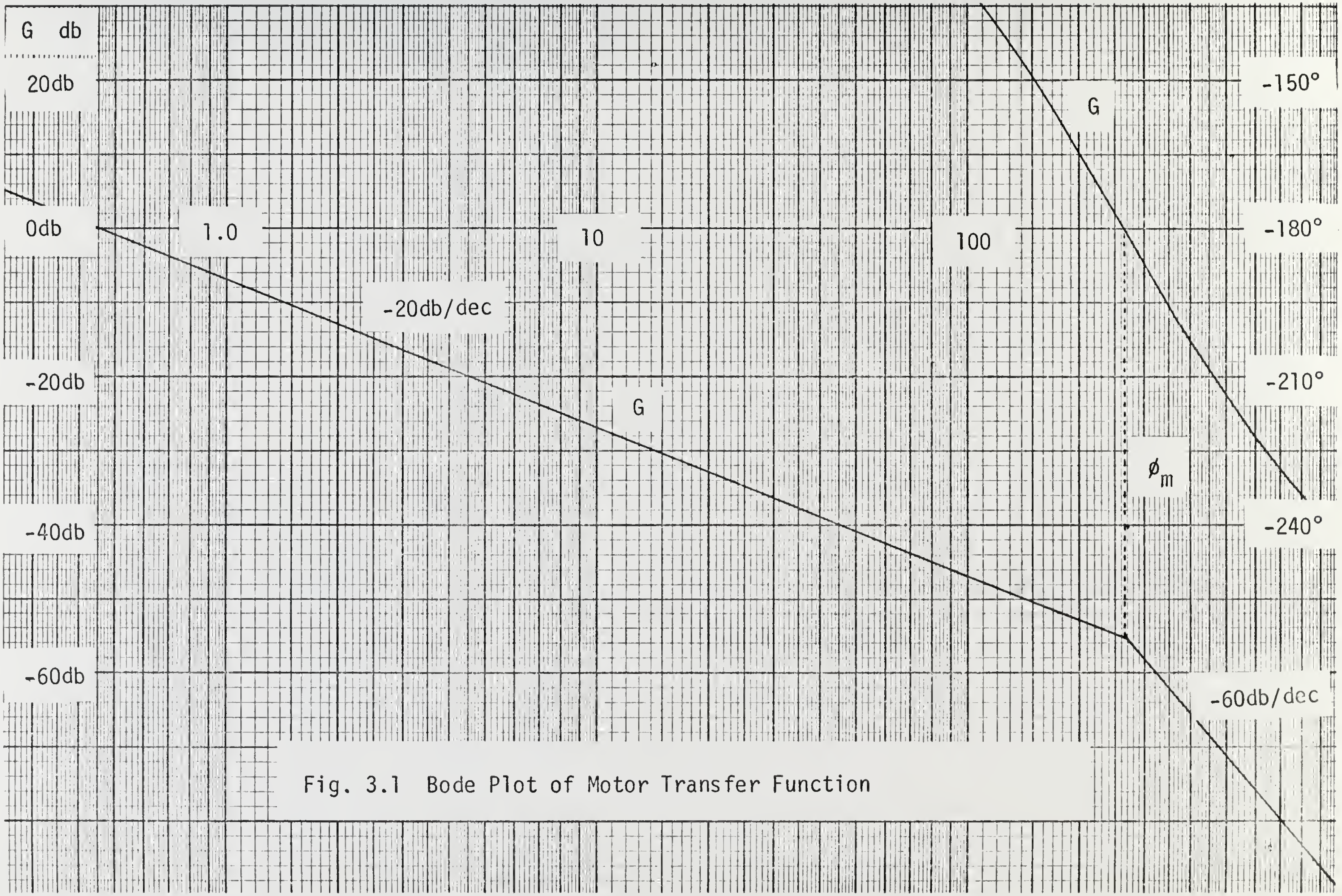

Fig. 3.1 Bode Plot of Motor Transfer Function

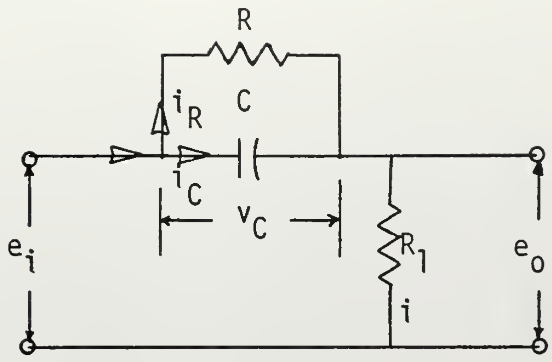

Fig. 3.2 Lead Filter Diagram

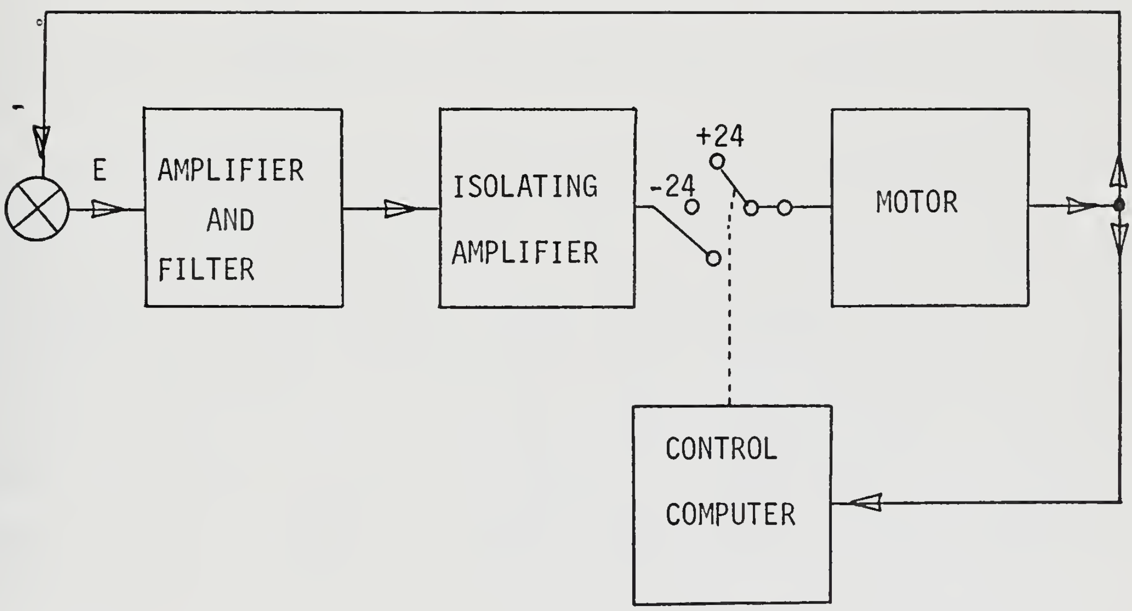

Fig. 3.3 Block Diagram of Implemented System

to the Bode plot, i.e.:

$$\alpha = \frac{\omega_1}{\omega_2} = \frac{R_1}{R + R_1}$$

In order to obtain the maximum phase correction with the filter, an α of 0.1 is used [3], and by trial and error process the system is compensated to provide the Bode plot shown in Fig. 3.4, the filter having the following characteristics

$$\alpha = .1$$

$$\tau = 0.004 \text{ sec}$$

Thus

$$G_c(s) = \frac{0.1(.004s + 1)}{.004s + 1} \tag{3.7}$$

The compensated system has now a transfer function

$$G(s) = \frac{5500(0.004s + 1)}{0.0004s + 1} \quad \frac{3432.3}{s(s^2 + 538.6s + 72741)} \tag{3.8}$$

It is a stable system with a phase margin of 52 degrees, its root locus plot has been presented in Fig. 3.5.

B. OBSERVATION OF THE EFFECT OF THE INITIAL CONDITIONS

With the help of the flexibility of CSMP, several values of initial conditions for the filter can be implemented in one computer run, thus allowing a gross estimation of the direction of the variation of the performance with the said initial condition.

The performance criterion used is that of the integral squared error in the linear mode. The optimum value of v_c is calculated to minimize this index. Since there are no weighting factors to consider in this

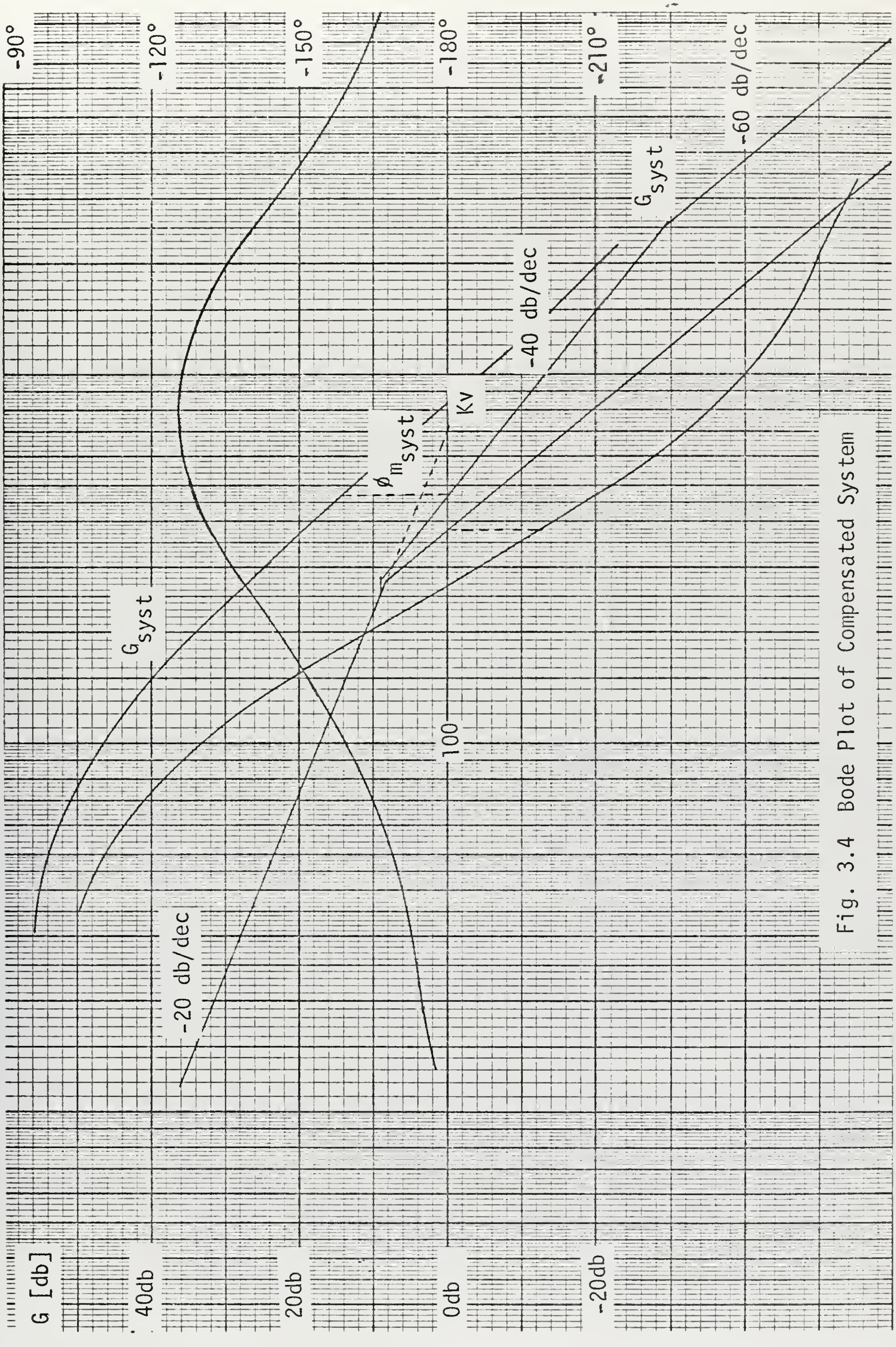

Fig. 3.4 Bode Plot of Compensated System

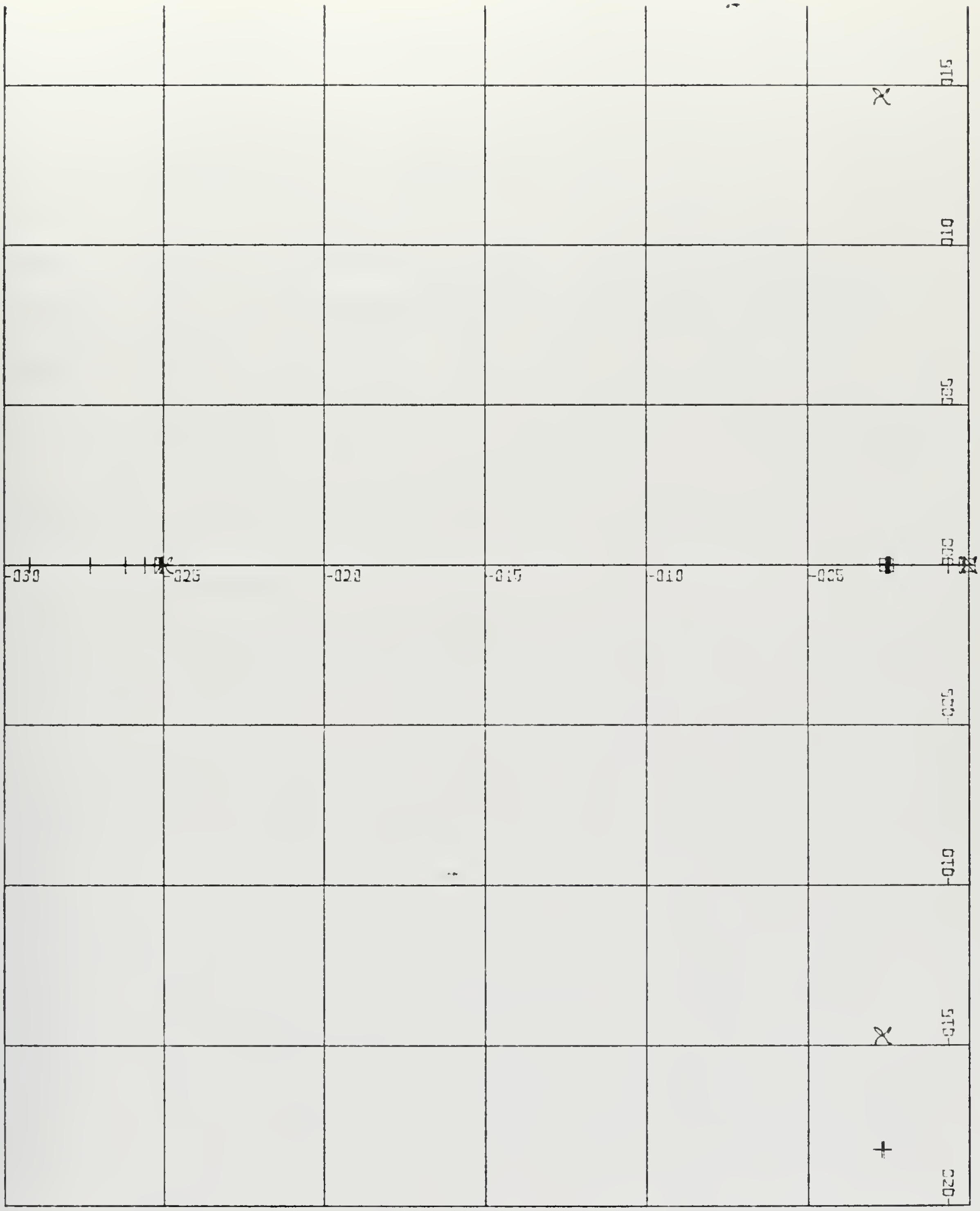

Fig. 3.5 Root-Locus Plot for the Compensated System

xscale = 500 units/inch
yscale = 5 units/inch

case, the problem is a very simple one, and the general expression for the optimal control

$$J = h(\underline{x}(t_f),t_f) + \int_{t_o}^{t_f} g(\underline{x}(t),\underline{u}(t),t)\,dt \tag{3.9}$$

where t_o and t_f are the initial and final time; h and g are scalar functions; $\underline{x}(t)$ is the trajectory and $\underline{u}(t)$ the control. [Ref. 4] reduces to simply

$$J = \int_{0}^{t_f} E\cdot E\,dt \tag{3.10}$$

in which

$$E = g(\underline{x}(t),u(t),t) \tag{3.11}$$

and is given by the transfer function and solved by the program itself.

Although flexible, CSMP places a number of restrictions on the use of subprograms and subroutines, therefore making it quite difficult to use the normal and known procedures for minimization and optimization. This leads to a rather crude method of determining the best suited initial condition for the filter, namely that of analyzing an ample range of values which show how the response of the system, and in particular the integral squared error, vary with the variable in question. Once a relative and feasible range of values for the initial condition is found, the best fit can be determined without much effort.

1. Algebraic Development

The solution for x(t) will now be pursued using the state variable form approach.

From the motor equations (3.1) and 3.2), it follows that:

$$\dddot{x}(t) = -a\dot{x}(t) - b\ddot{x}(t) + cv_a(t) \tag{3.12}$$

where $a = \frac{K_m K_b}{ML_a} + \frac{fR_a}{ML_a} = 72741$

$$b = \frac{f}{M} + \frac{R}{L_a} = 538.6$$

$$c = \frac{K_m}{ML_a} = 3432.3$$

Also, from the filter circuit in Fig. 3.2,

$$i(t) = i_R(t) + i_C(t) \tag{3.13}$$

$$\frac{e_o(t)}{R_1} = \frac{v_C(t)}{R} + C\,\dot{v}_C(t) \tag{3.14}$$

$$e_o(t) = e_i(t) + v_C(t) \tag{3.15}$$

Thus

$$\dot{v}_C(t) = -\left(\frac{1}{RC} + \frac{1}{R_1C}\right) v_C(t) + \frac{e_i(t)}{R_1C} \tag{3.16}$$

and since $RC = \tau$

and $R = 9R_1$

$$\dot{v}_C(t) = -\frac{10}{\tau}\, v_C(t) + \frac{9}{\tau}\, e_i(t) \tag{3.17}$$

This result holds under the assumption that the motor does not load the filter circuit, which can be achieved in practice with the use of an isolating amplifier. With this in mind, the analysis can be furthermore pursued to solve for the relationship between the output $x(t)$ and the initial condition on the capacitor $v_C(0)$.

Since the system is closed loop with zero input, the general expression

$$E = R - C$$

reduces to

$$E = - C$$

and therefore

$$e_i(t) = Ke(t) = - K\ x(t) \tag{3.18}$$

where K is the gain of the forward feed amplifier.

From (3.17) and (3.18)

$$\dot{v}_C(t) = - \frac{10}{\tau} v_C(t) - \frac{9K}{\tau} x(t) \tag{3.19}$$

Similarly, taking equations (3.12) and 3.15), and considering that

$$e_o(t) = v_a(t)$$

then:

$$\dddot{x}(t) = -cKx(t) - a\dot{x}(t) - b\ddot{x}(t) - cv_C(t) \tag{3.20}$$

Expressing equations 3.19 and 3.20 in their state variable form, and letting

$$x_1 = x(t)$$
$$x_2 = \dot{x}(t)$$
$$x_3 = \ddot{x}(t)$$
$$x_4 = v_C(t)$$

then

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) + \underline{B}\underline{u}(t) \tag{3.21}$$

where

$$\underline{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -cK & -a & -b & -c \\ -\frac{9K}{\tau} & 0 & 0 & -\frac{10}{\tau} \end{bmatrix} \quad \text{and } \underline{u}(t) = \underline{0}$$

Using Laplace transforms, the solution to equation (3.21) is

$$x(t) = \mathcal{L}^{-1}\,[sI - A]^{-1}x(0)$$

from which, the desired solution is the expression for $x_1 = x(t)$, that is

$$x_1(t) = \mathcal{L}^{-1}\left[\frac{1}{s(s^2+bs+a)(s+\frac{10}{\tau}) + cK(s+\frac{1}{\tau})}\quad (s+\frac{10}{\tau})(s^2+bs+a)x_1(0)\right.$$

$$\left. + (s+b)(s+\frac{10}{\tau})x_2(0) + (s+\frac{10}{\tau})x_3(0) + cx_4(0)\right]$$

$$x(t) = \mathcal{L}^{-1}\left[\frac{1}{(s+m)(s+n)[(s+p)^2+q^2]}\quad (s+\frac{10}{\tau})(s^2+bs+a)x(0)\right.$$

$$\left. + (s+b)(s+\frac{10}{\tau})\dot{x}(0) + (s+\frac{10}{\tau})\ddot{x}(0) + cv_C(0)\right]$$

where $m = 248.019$

$n = 2533.1968$

$p = 128.696$

$q = 241.98$

Thus

$$x(t) = [a_1x(0) + a_2\dot{x}(0) + a_3\ddot{x}(0) + a_4v_C(0)]e^{-mt}$$

$$+ [b_1x(0) + b_2\dot{x}(0) + b_3\ddot{x}(0) + b_4v_C(0)]e^{-nt}$$

$$+ [c_1x(0) + c_2\dot{x}(0) + c_3\ddot{x}(0) + c_4v_C(0)]e^{-pt}(\cos qt - \frac{p}{q} \sin qt)$$

$$+ [d_1x(0) + d_2\dot{x}(0) + d_3\ddot{x}(0) + d_4v_C(0)]e^{-pt}\sin qt \qquad (3.22)$$

with

$a_1 = 9.06\times10^{-3}$ $\qquad b_1 = 1.27\times10^{-2}$

$a_2 = 3.9\times10^{-3}$ $\qquad b_2 = -4.96\times10^{-6}$

$a_3 = 1.35\times10^{-6}$ $\qquad b_3 = 2.49\times10^{-9}$

$a_4 = -2.06\times10^{-4}$ $\qquad b_4 = 2.57\times10^{-7}$

$c_1 = 9.78\times10^{-1}$ $\qquad d_1 = 1.18$

$c_2 = -3.93\times10^{-3}$ $\qquad d_2 = 3.93\times10^{-3}$

$c_3 = -3.54\times10^{-6}$ $\qquad d_3 = -5.01\times10^{-7}$

$c_4 = 2.06\times10^{-4}$ $\qquad d_4 = 2.36\times10^{-4}$

2. Optimization of $v_C(0)$

The general expression for the Integral Squared Error is

$$ISE = \int_{t_o}^{t_f} (R-C)^2 \, dt \qquad (3.23)$$

that in this particular case, with $R = 0$ and $C = x$, reduces to

$$ISE = \int_{0}^{t_f} x^2(t) \, dt$$

To obtain a relative minimum of this function with respect to $v_C(0)$, its partial derivative with respect to the variable is made equal to zero, therefore

$$\frac{\partial ISE}{\partial v_C(0)} = \int_0^{t_f} 2\, x(t) \frac{\partial x(t)}{\partial v_C(0)}\, dt = 0 \qquad (3.24)$$

Thus, using equations (3.22) and (3.23)

$$\frac{\partial ISE}{\partial v_C(0)} = \int_0^{t_f} x(t)(a_4 e^{-mt} + b_4 e^{-nt} + c_4 e^{-pt}(\cos qt - \frac{p}{q}\sin qt) + d_4 e^{-pt}\sin qt)\, dt = 0$$

If t_f is taken large enough, say of the order of the settling time, the equation for $v_C(0)$ can be greatly simplified, since the exponential terms tend to zero. Hence the integration gives:

$$\begin{aligned} v_C(0) = {} & 25865.3(a_1 x(0) + a_2 \dot{x}(0) + z_3 \ddot{x}(0)) \\ & -14363.1(b_1 x(0) + b_2 \dot{x}(0) + b_3 \ddot{x}(0)) \\ & -29476.3(c_1 x(0) + c_2 \dot{x}(0) + c_3 \ddot{x}(0)) \end{aligned}$$

Taking typical values of initial conditions as those obtained from the computer simulation, viz.: $x(0) = -6.35\times10^{-5}$

$\dot{x}(0) = .27$

$\ddot{x}(0) = -190$

the optimum value of $v_C(0)$ is 79.3 volts, which corresponds to an initial condition of -79.3 volts. It will be shown later that this result agrees with the experimental values obtained from the computer.

To show that the inflection point of ISE vs $v_C(0)$ is in fact a minimum, the second derivative of the function is determined

$$\frac{\partial^2 ISE}{\partial v_C^{\,2}(0)} = 2 \quad (a_4 e^{-mt} + b_4 e^{-nt} + c_4 e^{-pt}(\cos qt - \frac{p}{q} \sin qt) + d_4 e^{-pt} \sin qt) \quad dt$$

$$= 4 \times 10^{-2}$$

A similar analysis could have been made to determine the optimum value of any other of the initial conditions or a combination of them.

The next section will be dedicated to analyze how the whole system is influenced by using different values of capacitor voltage.

IV. COMPUTER RESULTS

A. DETERMINATION OF INITIAL CONDITIONS

In order to have realistic valued initial conditions of the system when it enters the linear operational mode, a computer simulation of the bang-bang operation phase was accomplished. A convenient switching law was determined, such that for large enough errors, viz.: ten or more tracks, the motor was taken to around zero error with the appropriated final values of velocity and acceleration that would not produce a positive or negative overshoot when the loop was closed.

It is important to note here that if the arm override enters a neighboring track, a new control law is encountered, and thus the system is driven completely out and away from its destination track. Therefore, it is imperative that once the head has entered the desired track zone it does not leave it, and if it does, it is only by a very small and insignificant distance (one mil or so), so that the inertia of the system would take care of the override, not allowing the head to be driven off.

With this in mind, the results obtained from the discontinuous operation mode were tried as initial conditions on the closed loop mode. The width of the track considered was of 5 mils or 12.7×10^{-2} mm.

Once a set of velocity and acceleration initial conditions were proved satisfactory to keep the head within the destination track, a rounded variation about these values was tried out in order to determine the permissible amount of error in these parameters in entering the linear zone. It should be remembered that there is no accurate means

to determine velocity or acceleration, hence the need of having a possible wide range of these values at the entering point, and a control law that is based on position alone.

A computer program for the discontinuous phase is presented in the appendix, the typical phase plane plots for this mode being shown on Fig. 4.1.

B. TYPICAL PHASE PLANE RESULTS OF CONTINUOUS SYSTEM

The computer program used in this part of the experiment is further explained in the appendix. More details on the programming method can be found in Ref. [5].

The simulation proved satisfactory to show the way the response of the system varies with the initial condition on the filter capacitor. By varying this initial condition over a wide range of values, the integral squared error as a function of the capacitor voltage was plotted, and the resulting curves are shown on Fig. 4.2. It can be seen from this figure that the integral squared error does minimize within the range of practicable voltages for the capacitor. Also, from the phase plane plots shown on Fig. 4.3, and the transient response on Fig. 4.4, the effect of the initial condition proves to be of great importance in obtaining the solution to the problem at hand. A zero initial condition turned out to be far from ideal in this case, and the voltage derived in the analytic discussion showed to be the best solution in practice.

To have a better idea of the transient response of the system, a computer run for a unit step input and zero initial conditions was made, the position vs. time curve being shown on Fig. 4.5. By a comparison

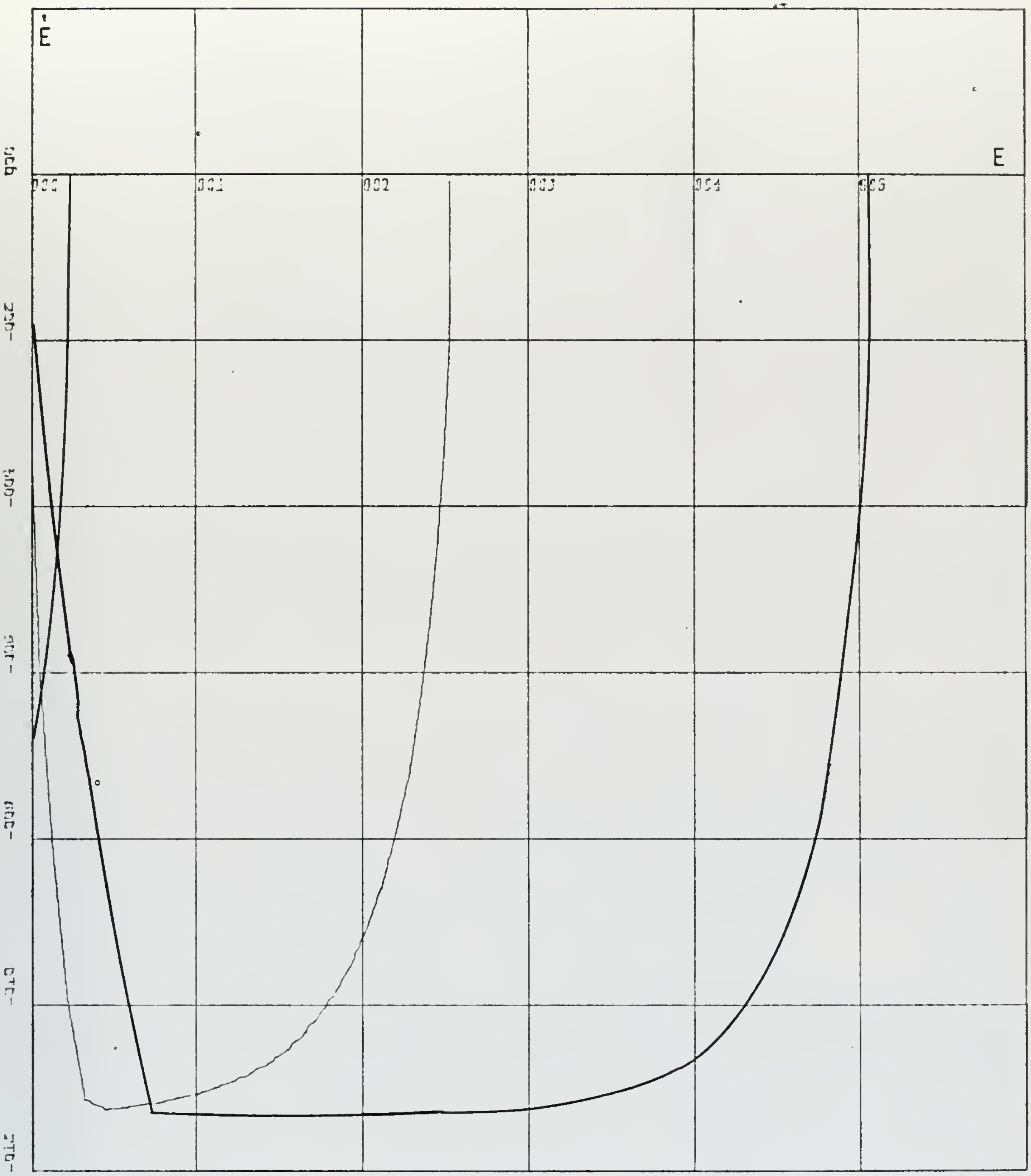

Fig. 4.1 Phase plane plots for discontinuous operating mode of system. The three curves show different values of positioning orders; the largest used was 400 tracks, the smallest 20 tracks. Switching law used is a linear one, namely E = INITIAL ERROR/7.
x scale = .01 units/inch
y scale = 0.2 units/inch

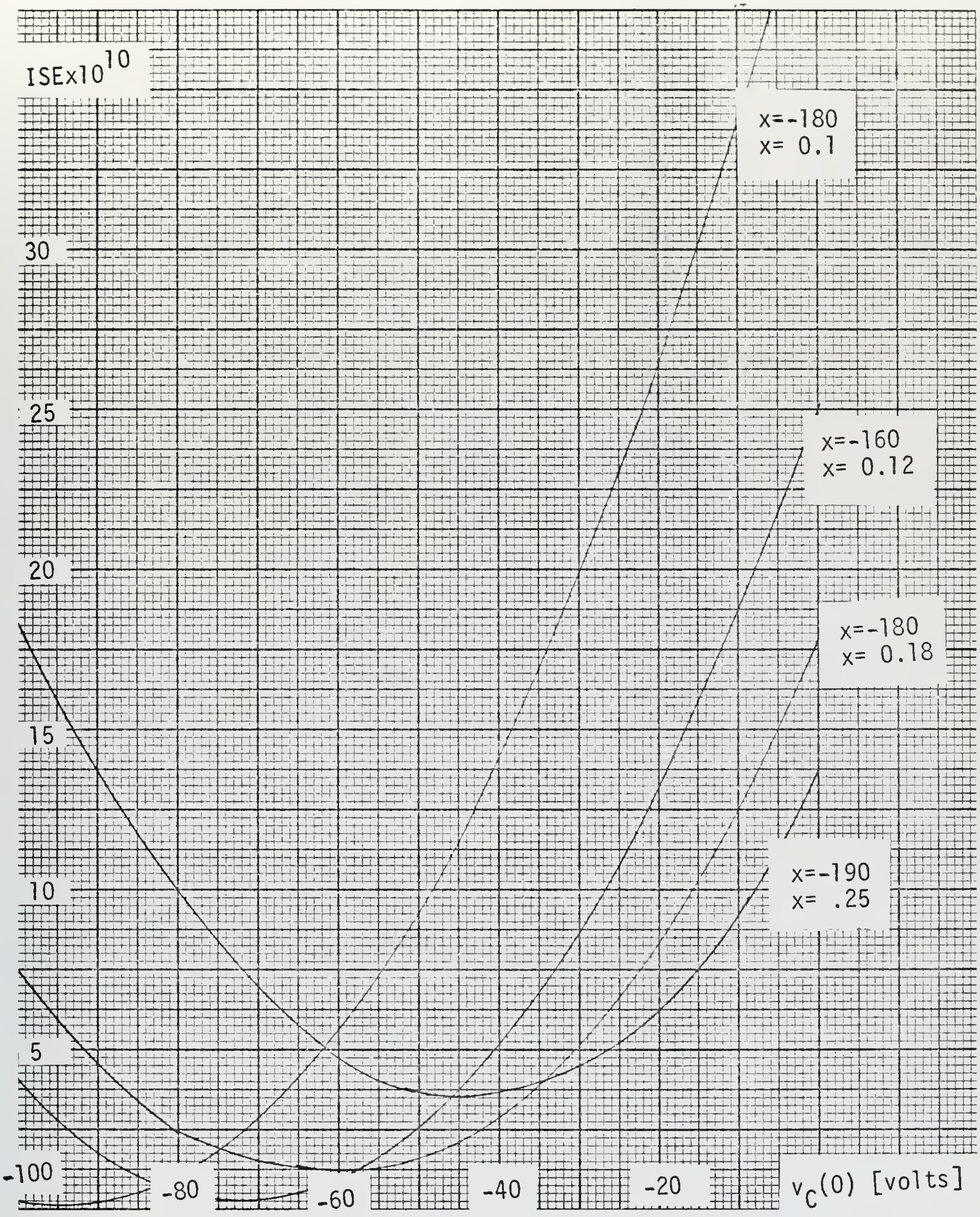

Fig. 4.2 Plot of integral squared error vs. filter initial condition for different pairs of velocity and acceleration values. Note that the optimum value for $v_C(0)$ with the given $\ddot{x}(t)$ and $\dot{x}(t)$ is around -60 volts.

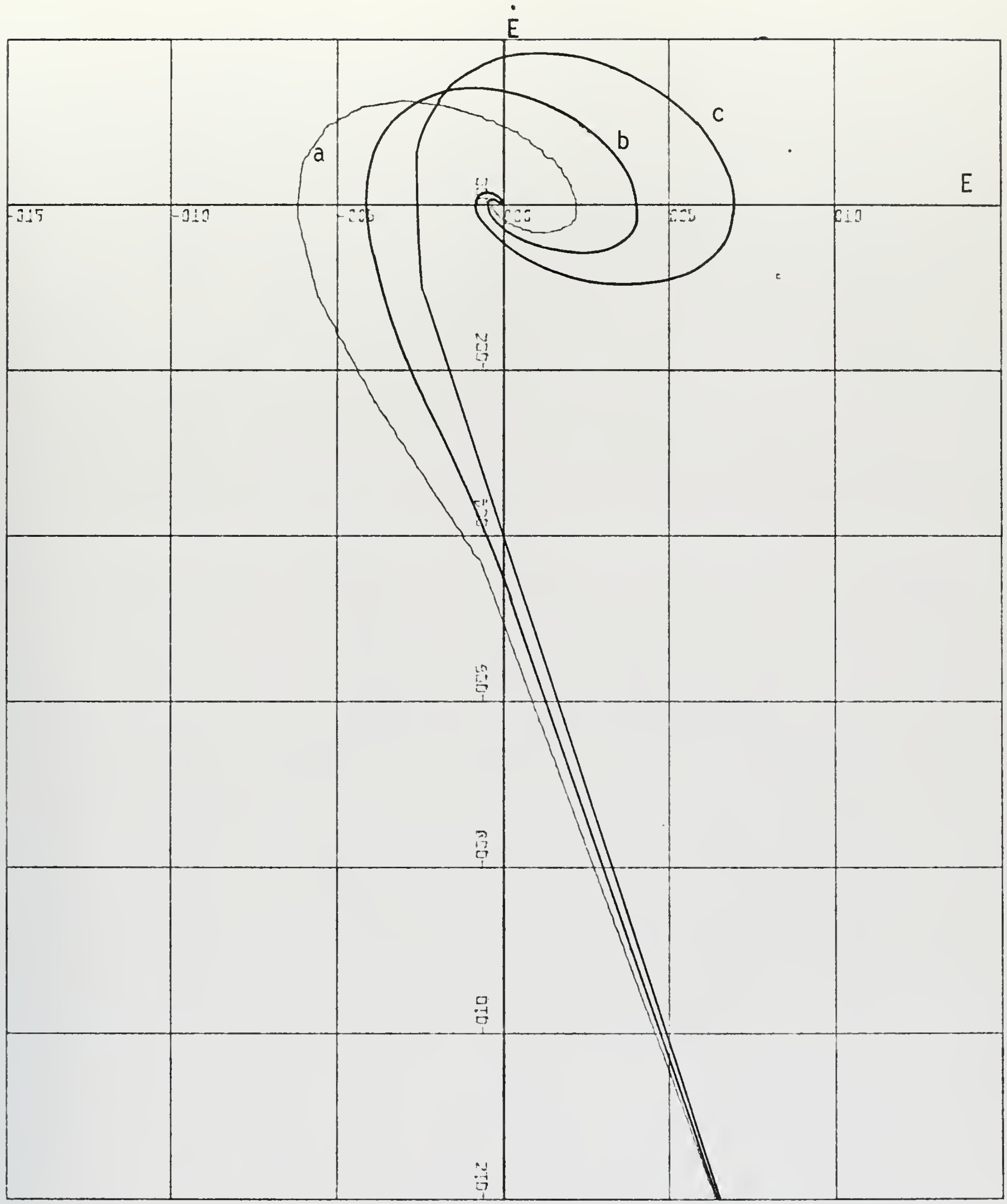

Fig. 4.3 $\dot{E}$ vs E for $\ddot{x}(0)=-160$ m/sec^2 and $\dot{x}(0)=.12$ m/sec.
Curve (a) shows response for $v_C(0)=-90$ volts,
(b) for $v_C(0)=-85$ volts, and
(c) for $v_C(0)=-80$ volts.

x scale = .02 m per inch
y scale = 5×10^{-5} m/seg per inch

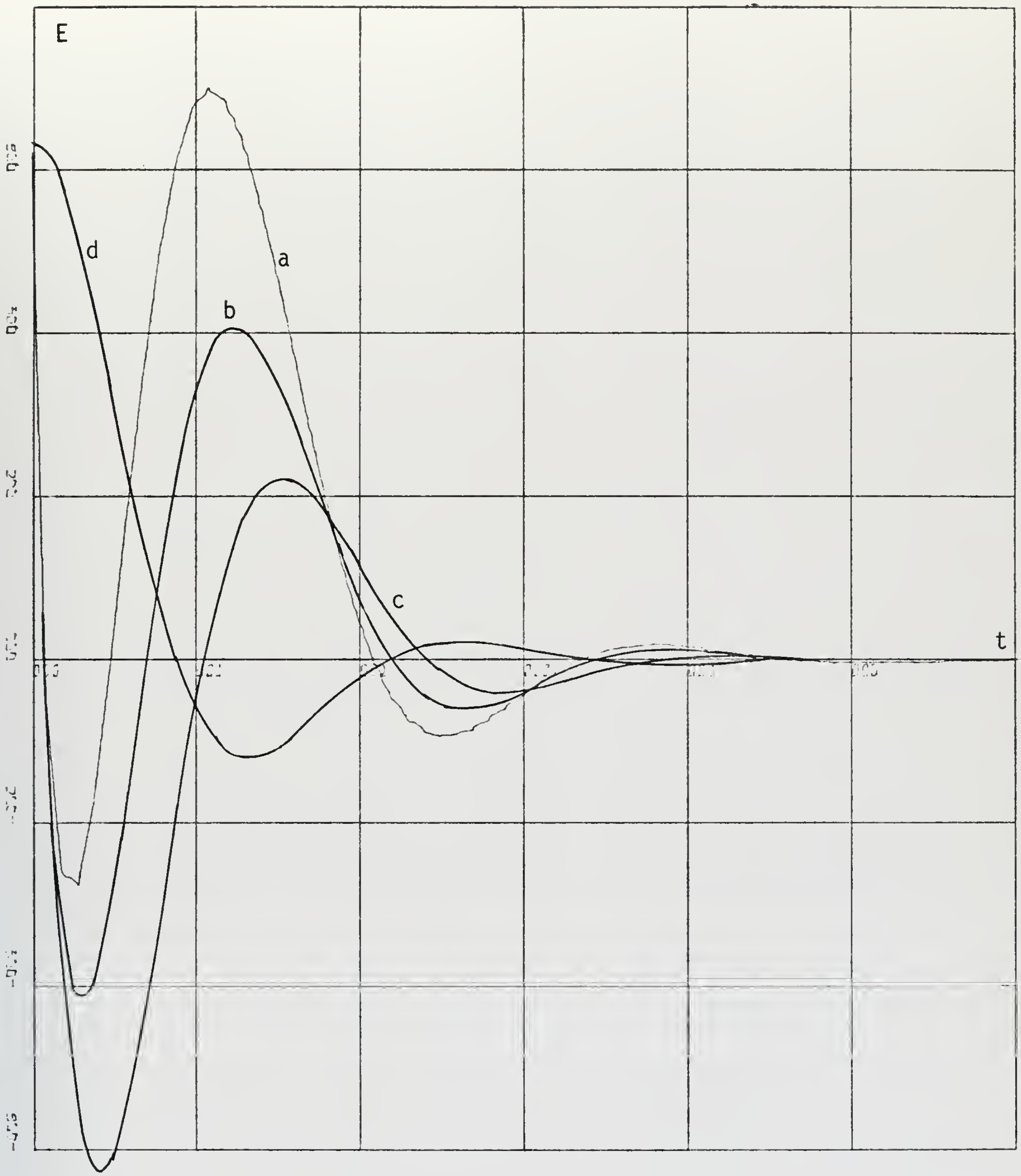

Fig. 4.4 E vs t plot for curyes shown on Fig. 4.3. Curves (a) (b) and (c) show the transient response for $v_C(0)$=-90, -85, and -80 volts respectively. Curve (d) has been added to show the transient response with zero velocity and acceleration, $v_C(0)$=0 in this case.

x scale = .01 sec per inch
y scale = $2x.0^{-5}$ m per inch

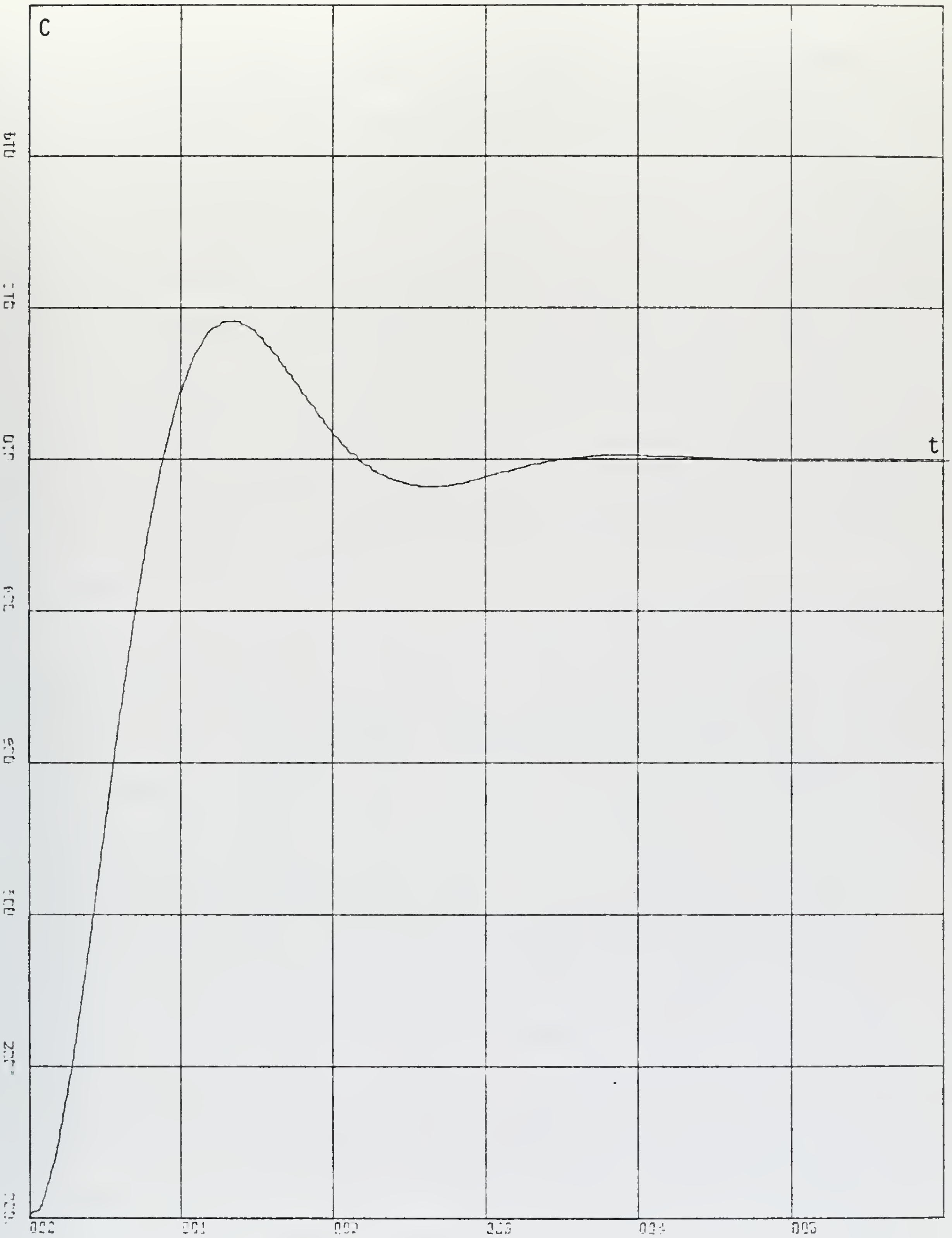

Fig. 4.5 Step Input Response of the System.

x scale = .01 sec per inch
y scale = .2 m per inch

of this curve with the ones obtained as solutions to the discontinuous problem, it can be seen that the initial conditions on velocity and acceleration greatly affect the behaviour of the system, making its stability quite critical depending on their values. To control this situation, the initial condition on the filter capacitor has to be set to an appropriate value which is dependent on the velocity and acceleration of the motor at the point of entry to the linear mode. Therefore, the best possible solution would be one that used a different value of capacitor voltage for each set of initial conditions at this point. Again, this cannot be done because of the lack of information on these values, thus leading to the use of a best fit voltage that will apply to a whole set of parameters.

It has been the intention to show here, with the help of the phase plane plots, of how the error varies with the initial condition on the capacitor. The various computer runs showed that, although this voltage could prevent a negative overshoot of the error at the beginning of the trajectory, it produced a positive one later on if not chosen properly, thus providing an undesired solution. In this respect, the integral squared error was very efficient in detecting these effects, and although with the specified initial conditions of acceleration and velocity in some cases the system did overshoot and left the bounded region, using the initial condition on the filter as indicated by the minimized integral squared error, the reader head was maintained within the specified limits. Fig. 4.6 shows the difference of the trajectories and the overshoot caused by the use of a wrong voltage, and how it is remedied with an adequate one. The values of the error when overriding were of the order of 0.7 to 0.9 mm., and were conveniently reduced to

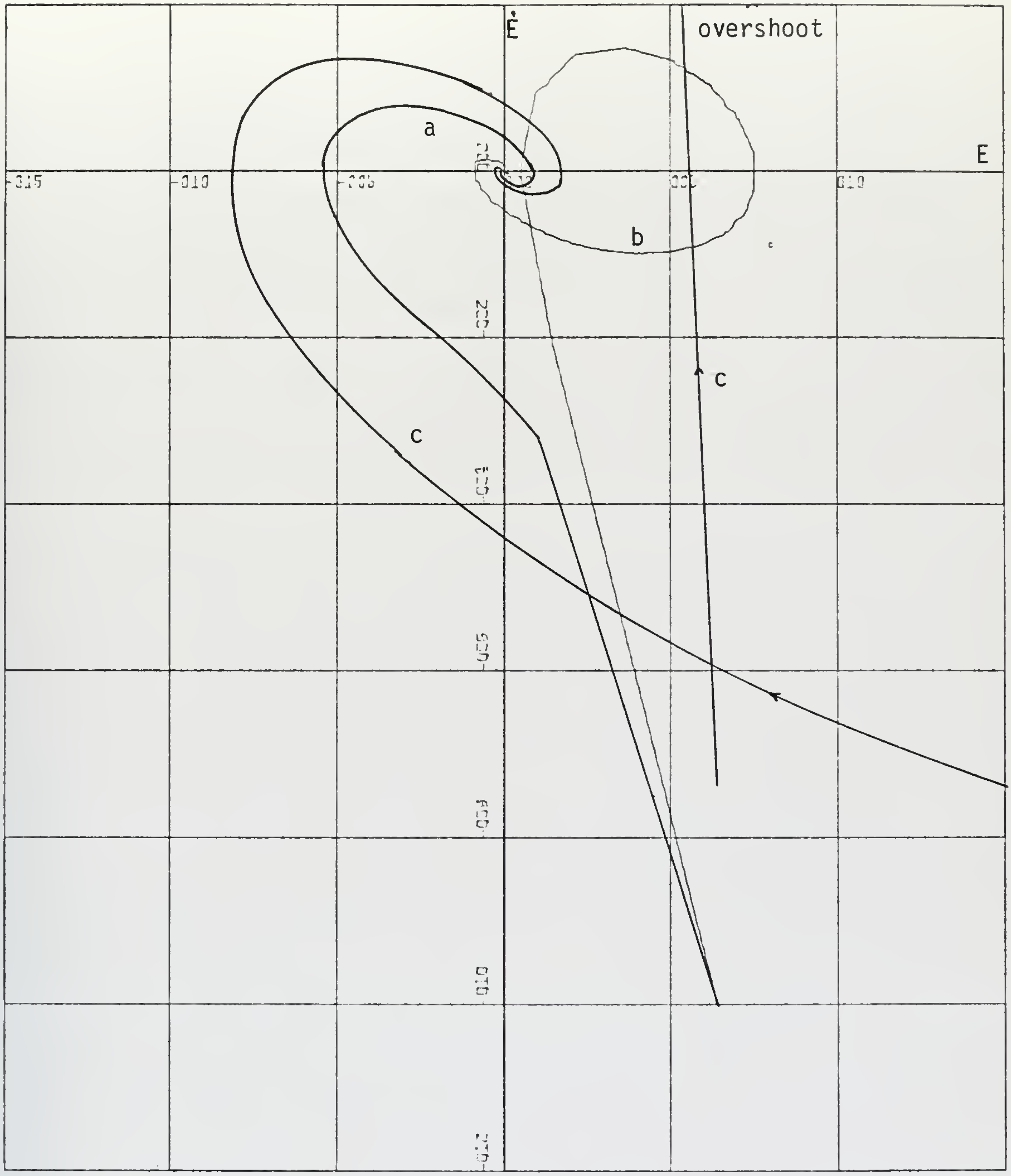

Fig. 4.6 E vs E curves for $\ddot{x}(0)=-180$ m/sec^2 and $\dot{x}(0)=.10$ m/sec. Curve (a) is for a $v_C(0)=-100$ volts, (b) for $v_C(0)=-85$ volts. Notice in this curve that although there is a tendency to override because of a high speed of entry, the system is kept within the bounded region by the adequate use of $v_C(0)=0$. Curve (c) shows an overshoot caused by the use of $v_C(0)=0$.

x scale = 5×10^{-5} m per inch y scale = .02 m/sec per inch

0.05 or 0.04. The integral squared error was reduced by a factor which ranges between 20 and 300, which means that the performance of the whole system is noticeably improved.

Finally, it is important to point out from the experimental data, that when reducing the integral squared error, not only does the trajectory stay inside the bounded region, but also the settling time is minimum for this case.

V. CONCLUSION

The special case presented here has served to show the significance of the initial condition of a filter when connected to a system to compensate it. In practice, other types of filters can be used, and a different type of feed encountered, but the analysis will be similar, and the effects of the initial condition should be carefully studied in order to obtain an optimized control.

The bounded characteristics of this system are evidently no restrictions for application of the criterion here presented. It is generally desired to achieve minimum time of response, thus making the assumptions true for all cases. More sophisticated types of control can be used based on the results obtained, such as adjusting the initial condition according with the values of other parameters of the system if these are well known, or other convenient control that, without affecting cost or other factors, will provide a significant improvement on system performance.

Thus, the main idea introduced by this work is that when using a compensator in a discontinuous manner, initial conditions of the system play an important role in its response characteristics, and should therefore be considered carefully to obtain the best possible solution. The specific control law used must be determined for each particular case.

APPENDIX A

Explanation of Simulation

S/360 CSMP is a program for the simulation of continuous systems, hence limiting the possibility of simulating the complete discontinuous problem in one single program when a filter with initial condition is added to the diagram after time =0. This leads to the use of two separate programs in which, the initial conditions of the second are the final values of the first.

In order to account for the initial conditions as specified by the problem at hand, some of the operations had to be made longer, as is the case of the three operations involved in solving the filter equations instead of just using the LEDLAG statement provided by CSMP, and the two separate integrations for the motor equations in place of the CMPXPL statement.

The program provides an application-oriented input language that accepts problems expressed in the form of either an analog block diagram or a system of ordinary differential equations. The NOSORT statement in the discontinuous mode program allows the use of logical statements as the ones known in Fortran.

```
      LABEL  MULTIMODE AUTOMATIC CONTROL SYSTEM

DISCONTINUOUS OPERATION PHASE

      PARAM  KM = 3432.3, ZETA = .9985, OMEGA = 269.7
      PARAM  K = 55000, OMSQ = 72740.95, TAU = .004
      PARAM  EINIC = (.02544,.00254)

 DYNAMIC

   NOSORT

      INPUT = 24

      IF ( E.LE.EINIC/7 ) INPUT = -24

SOLUTION TO INSTRUMENT SERVO EQUATIONS

      C3DOT = AUX1 + AUX2 + AUX3
      C2DOT = INTGRL(0.,C3DOT)
      CDOT = INTGRL(0.,C2DOT)
      C = INTGRL(0.,CDOT)
      AUX1 = -2 * ZETA * OMEGA * C2DOT
      AUX2 = -OMSQ * CDOT
      AUX3 = KM * INPUT
      EDOT = -CDOT
      E = EINIC - C

FINISH STATEMENT TERMINATES THE INTEGRATION FOR
SPECIFIED VALUE OF CDOT

      FINISH CDOT = -.001

PRINTOUT OF THE INDICATED VARIABLES

      PRINT  C3DOT,C2DOT,CDOT,C,E

DATA FOR DRAW SUBROUTINE

      PREPARE E,EDOT

      TIMER FINTIM = .1, DELT = .001, PRDEL = .001

      END
      STOP
ENDJOB
```

```
      LABEL  MULTIMODE AUTOMATIC CONTROL SYSTEM

CLOSED LOOP, LINEAR OPERATION PHASE

      PARAM  KM = 3432.3, ZETA = .9985, OMEGA = 269.7
      PARAM  K = 55000, OMSQ = 72740.95, TAU = .004
      INCON CO = -6.35E-05
      INCON C2DOTO = -160, CDOTO = .08, VCO=( -100,-85,0 )
 DYNAMIC

      EPRIME = E * K

SOLUTION TO FILTER EQUATIONS

VC IS THE VOLTAGE ACROSS THE CAPACITOR
EC IS THE OUTPUT OF THE COMPENSATOR NETWORK

      VCDOT = -( 10/TAU ) * VC + ( 9/TAU ) * EPRIME
      VC = INTGRL(VCO,VCDOT)
      EC = EPRIME - VC

SOLUTION TO INSTRUMENT SERVO EQUATIONS

      C3DOT = AUX1 + AUX2 + AUX3
      C2DOT = INTGRL(C2DOTO,C3DOT)
      CDOT = INTGRL(CDOTO,C2DOT)
      C = INTGRL(CO,CDOT)
      AUX1 = -2 * ZETA * OMEGA * C2DOT
      AUX2 = -OMSQ * CDOT
      AUX3 = KM * EC

      E2DOT =- C2DOT
      EDOT = - CDOT
      E = - C

EVALUATION OF THE INTEGRAL SQUARED ERROR

      SQER = E * E * 1.0E 10
      INSQER = INTGRL(0.,SQER)

PRINTOUT OF THE INDICATED VARIABLES

      PRINT E2DOT, EDOT,E,INSQER

DATA FOR DRAW SUBROUTINE TO OBTAIN GRAPHICAL OUTPUT
OF E VS. EDOT

      PREPARE E,EDOT

      TIMER FINTIM = .1, DELT = .001, PRDEL = .001

      END
      STOP
ENDJOB
```

CSMP

TIME	E2DOT	EDOT	E	INSQER
0.0	1.6000E 02	-8.0000E-02	6.3500E-05	0.0
1.0000E-03	9.1178E 01	4.2633E-02	5.0477E-05	2.2244E-02
2.0000E-03	4.4269E 01	1.0916E-01	1.3027E-04	9.9713E-02
3.0000E-03	1.0041E 01	1.3536E-01	2.5538E-04	4.7640E-01
4.0000E-03	-1.3587E 01	1.3277E-01	3.9142E-04	1.5392E 00
5.0000E-03	-2.8262E 01	1.1117E-01	5.1461E-04	3.6205E 00
6.0000E-03	-3.5644E 01	7.8680E-02	6.1015E-04	6.8214E 00
7.0000E-03	-3.7365E 01	4.1770E-02	6.7052E-04	1.0964E 01
8.0000E-03	-3.4956E 01	5.3252E-03	6.9386E-04	1.5660E 01
9.0000E-03	-2.9794E 01	-2.7227E-02	6.8248E-04	2.0433E 01
1.0000E-02	-2.3063E 01	-5.3742E-02	6.4143E-04	2.4846E 01
1.1000E-02	-1.5734E 01	-7.3156E-02	5.7737E-04	2.8583E 01
1.2000E-02	-8.5570E 00	-8.5263E-02	4.9756E-04	3.1488E 01
1.3000E-02	-2.0755E 00	-9.0504E-02	4.0914E-04	3.3554E 01
1.4000E-02	3.3608E 00	-8.9764E-02	3.1855E-04	3.4884E 01
1.5000E-02	7.5678E 00	-8.4194E-02	2.3122E-04	3.5643E 01
1.6000E-02	1.0498E 01	-7.5056E-02	1.5135E-04	3.6011E 01
1.7000E-02	1.2210E 01	-6.3605E-02	8.1881E-05	3.6149E 01
1.8000E-02	1.2836E 01	-5.0999E-02	2.4527E-05	3.6179E 01
1.9000E-02	1.2560E 01	-3.8234E-02	-2.0066E-05	3.6181E 01
2.0000E-02	1.1584E 01	-2.6112E-02	-5.2157E-05	3.6195E 01
2.1000E-02	1.0119E 01	-1.5228E-02	-7.2705E-05	3.6236E 01
2.2000E-02	8.3597E 00	-5.9720E-03	-8.3158E-05	3.6298E 01
2.3000E-02	6.4805E 00	1.4516E-03	-8.5262E-05	3.6370E 01
2.4000E-02	4.6254E 00	6.9974E-03	-8.0883E-05	3.6440E 01
2.5000E-02	2.9058E 00	1.0748E-02	-7.1867E-05	3.6499E 01
2.6000E-02	1.3999E 00	1.2881E-02	-5.9927E-05	3.6542E 01
2.7000E-02	1.5505E-01	1.3635E-02	-4.6565E-05	3.6571E 01
2.8000E-02	-8.0851E-01	1.3285E-02	-3.3025E-05	3.6587E 01
2.9000E-02	-1.4932E 00	1.2111E-02	-2.0270E-05	3.6594E 01
3.0000E-02	-1.9187E 00	1.0385E-02	-8.9864E-06	3.6596E 01
3.1000E-02	-2.1172E 00	8.3496E-03	3.9736E-07	3.6596E 01
3.2000E-02	-2.1279E 00	6.2132E-03	7.6796E-06	3.6597E 01
3.3000E-02	-1.9935E 00	4.1422E-03	1.2846E-05	3.6598E 01
3.4000E-02	-1.7564E 00	2.2603E-03	1.6027E-05	3.6600E 01
3.5000E-02	-1.4559E 00	6.5040E-04	1.7458E-05	3.6603E 01
3.6000E-02	-1.1266E 00	-6.4198E-04	1.7434E-05	3.6606E 01
3.7000E-02	-7.9684E-01	-1.6027E-03	1.6285E-05	3.6609E 01
3.8000E-02	-4.8874E-01	-2.2429E-03	1.4336E-05	3.6611E 01
3.9000E-02	-2.1780E-01	-2.5926E-03	1.1896E-05	3.6613E 01
4.0000E-02	6.4451E-03	-2.6942E-03	9.2336E-06	3.6614E 01
4.1000E-02	1.7974E-01	-2.5968E-03	6.5737E-06	3.6614E 01
4.2000E-02	3.0215E-01	-2.3517E-03	4.0893E-06	3.6615E 01
4.3000E-02	3.7712E-01	-2.0083E-03	1.9030E-06	3.6615E 01
4.4000E-02	4.1042E-01	-1.6114E-03	9.0410E-08	3.6615E 01
4.5000E-02	4.0928E-01	-1.1990E-03	-1.3147E-06	3.6615E 01
4.6000E-02	3.8157E-01	-8.0167E-04	-2.3127E-06	3.6615E 01
4.7000E-02	3.3511E-01	-4.4209E-04	-2.9307E-06	3.6615E 01
4.8000E-02	2.7720E-01	-1.3526E-04	-3.2145E-06	3.6615E 01
4.9000E-02	2.1424E-01	1.1064E-04	-3.2216E-06	3.6615E 01
5.0000E-02	1.5150E-01	2.9331E-04	-3.0144E-06	3.6615E 01
5.1000E-02	9.3059E-02	4.1509E-04	-2.6553E-06	3.6615E 01
5.2000E-02	4.1778E-02	4.8183E-04	-2.2026E-C6	3.6615E 01
5.3000E-02	-5.9734E-04	5.0163E-04	-1.7073E-06	3.6615E 01
5.4000E-02	-3.3303E-02	4.8386E-04	-1.2118E-06	3.6615E 01
5.5000E-02	-5.6383E-02	4.3824E-04	-7.4885E-07	3.6615E 01

RKS INTEGRATION VCO = 0.0

CSMP

TIME	E2DOT	EDOT	E	INSQER
0.0	1.6000E 02	-8.0000E-02	6.3500E-05	0.0
1.0000E-03	6.1846E 00	-2.7655E-02	2.1242E-05	1.6187E-02
2.0000E-03	-9.0044E-01	-2.7320E-02	-5.7541E-06	1.7391E-02
3.0000E-03	2.1396E 00	-2.6765E-02	-3.3057E-05	2.1798E-02
4.0000E-03	4.6667E 00	-2.3266E-02	-5.8283E-05	4.3457E-02
5.0000E-03	6.0623E 00	-1.7813E-02	-7.8939E-05	9.1515E-02
6.0000E-03	6.5067E 00	-1.1459E-02	-9.3612E-05	1.6705E-01
7.0000E-03	6.2351E 00	-5.0381E-03	-1.0184E-04	2.6365E-01
8.0000E-03	5.4687E 00	8.4653E-04	-1.0387E-04	3.7046E-01
9.0000E-03	4.4023E 00	5.7997E-03	-1.0046E-04	4.7569E-01
1.0000E-02	3.1991E 00	9.6059E-03	-9.2654E-05	5.6958E-01
1.1000E-02	1.9890E 00	1.2196E-02	-8.1652E-05	6.4601E-01
1.2000E-02	8.6843E-01	1.3614E-02	-6.8653E-05	7.0281E-01
1.3000E-02	-9.7302E-02	1.3985E-02	-5.4773E-05	7.4109E-01
1.4000E-02	-8.7046E-01	1.3484E-02	-4.0975E-05	7.6413E-01
1.5000E-02	-1.4366E 00	1.2313E-02	-2.8029E-05	7.7611E-01
1.6000E-02	-1.7998E 00	1.0679E-02	-1.6503E-05	7.8112E-01
1.7000E-02	-1.9779E 00	8.7753E-03	-6.7610E-06	7.8251E-01
1.8000E-02	-1.9975E 00	6.7757E-03	1.0161E-06	7.8264E-01
1.9000E-02	-1.8908E 00	4.8224E-03	6.8062E-06	7.8283E-01
2.0000E-02	-1.6913E 00	3.0250E-03	1.0713E-05	7.8364E-01
2.1000E-02	-1.4316E 00	1.4599E-03	1.2934E-05	7.8507E-01
2.2000E-02	-1.1411E 00	1.7209E-04	1.3726E-05	7.8687E-01
2.3000E-02	-8.4496E-01	-8.2050E-04	1.3377E-05	7.8873E-01
2.4000E-02	-5.6319E-01	-1.5227E-03	1.2182E-05	7.9038E-01
2.5000E-02	-3.1045E-01	-1.9566E-03	1.0421E-05	7.9167E-01
2.6000E-02	-9.6309E-02	-2.1565E-03	8.3466E-06	7.9256E-01
2.7000E-02	7.4261E-02	-2.1638E-03	6.1723E-06	7.9309E-01
2.8000E-02	2.0021E-01	-2.0229E-03	4.0684E-06	7.9335E-01
2.9000E-02	2.8360E-01	-1.7776E-03	2.1612E-06	7.9345E-01
3.0000E-02	3.2878E-01	-1.4684E-03	5.3445E-07	7.9347E-01
3.1000E-02	3.4158E-01	-1.1308E-03	-7.6625E-07	7.9347E-01
3.2000E-02	3.2861E-01	-7.9389E-04	-1.7275E-06	7.9349E-01
3.3000E-02	2.9666E-01	-4.7995E-04	-2.3618E-06	7.9353E-01
3.4000E-02	2.5220E-01	-2.0473E-04	-2.7004E-06	7.9360E-01
3.5000E-02	2.0105E-01	2.2232E-05	-2.7874E-06	7.9368E-01
3.6000E-02	1.4814E-01	1.9680E-04	-2.6735E-06	7.9375E-01
3.7000E-02	9.7371E-02	3.1924E-04	-2.4112E-06	7.9382E-01
3.8000E-02	5.1630E-02	3.9323E-04	-2.0512E-06	7.9387E-01
3.9000E-02	1.2804E-02	4.2481E-04	-1.6389E-06	7.9390E-01
4.0000E-02	-1.8106E-02	4.2148E-04	-1.2132E-06	7.9392E-01
4.1000E-02	-4.0851E-02	3.9133E-04	-8.0489E-07	7.9393E-01
4.2000E-02	-5.5777E-02	3.4239E-04	-4.3679E-07	7.9393E-01
4.3000E-02	-6.3670E-02	2.8212E-04	-1.2388E-07	7.9394E-01
4.4000E-02	-6.5600E-02	2.1704E-04	1.2587E-07	7.9394E-01
4.5000E-02	-6.2789E-02	1.5250E-04	3.1040E-07	7.9394E-01
4.6000E-02	-5.6496E-02	9.2623E-05	4.3244E-07	7.9394E-01
4.7000E-02	-4.7922E-02	4.0271E-05	4.9817E-07	7.9394E-01
4.8000E-02	-3.8146E-02	-2.8222E-06	5.1608E-07	7.9394E-01
4.9000E-02	-2.8084E-02	-3.5928E-05	4.9587E-07	7.9394E-01
5.0000E-02	-1.8461E-02	-5.9138E-05	4.4753E-07	7.9395E-01
5.1000E-02	-9.8099E-03	-7.3175E-05	3.8065E-07	7.9395E-01
5.2000E-02	-2.4792E-03	-7.9200E-05	3.0386E-07	7.9395E-01
5.3000E-02	3.3490E-03	-7.8636E-05	2.2445E-07	7.9395E-01
5.4000E-02	7.6326E-03	-7.3018E-05	1.4827E-07	7.9395E-01
5.5000E-02	1.0440E-02	-6.3864E-05	7.9593E-08	7.9395E-01

RKS INTEGRATION VCO =-1.0000E 02

CSMP

TIME	E2DOT	EDOT	E	INSQER
0.0	1.6000E 02	-8.0000E-02	6.3500E-05	0.0
1.0000E-03	1.8934E 01	-1.7112E-02	2.5627E-05	1.6912E-02
2.0000E-03	5.8752E 00	-6.8480E-03	1.4650E-05	2.0742E-02
3.0000E-03	3.3248E 00	-2.4458E-03	1.0209E-05	2.2214E-02
4.0000E-03	1.9287E 00	1.3971E-04	9.1718E-06	2.3113E-02
5.0000E-03	9.1371E-01	1.5348E-03	1.0094E-05	2.4020E-02
6.0000E-03	1.8405E-01	2.0618E-03	1.1953E-05	2.5228E-02
7.0000E-03	-3.0491E-01	1.9831E-03	1.4016E-05	2.6919E-02
8.0000E-03	-5.9500E-01	1.5183E-03	1.5791E-05	2.9155E-02
9.0000E-03	-7.2719E-01	8.4570E-04	1.6984E-05	3.1860E-02
1.0000E-02	-7.4023E-01	1.0360E-04	1.7459E-05	3.4847E-02
1.1000E-02	-6.6937E-01	-6.0684E-04	1.7202E-05	3.7871E-02
1.2000E-02	-5.4539E-01	-1.2175E-03	1.6279E-05	4.0691E-02
1.3000E-02	-3.9403E-01	-1.6885E-03	1.4814E-05	4.3122E-02
1.4000E-02	-2.3577E-01	-2.0033E-03	1.2955E-05	4.5060E-02
1.5000E-02	-8.5958E-02	-2.1630E-03	1.0859E-05	4.6485E-02
1.6000E-02	4.4837E-02	-2.1816E-03	8.6757E-06	4.7443E-02
1.7000E-02	1.5027E-01	-2.0818E-03	6.5352E-06	4.8024E-02
1.8000E-02	2.2757E-01	-1.8905E-03	4.5427E-06	4.8332E-02
1.9000E-02	2.7678E-01	-1.6360E-03	2.7753E-06	4.8467E-02
2.0000E-02	3.0006E-01	-1.3456E-03	1.2826E-06	4.8509E-02
2.1000E-02	3.0101E-01	-1.0434E-03	8.8049E-08	4.8515E-02
2.2000E-02	2.8404E-01	-7.4953E-04	-8.0697E-07	4.8517E-02
2.3000E-02	2.5386E-01	-4.7968E-04	-1.4191E-06	4.8530E-02
2.4000E-02	2.1511E-01	-2.4466E-04	-1.7780E-06	4.8556E-02
2.5000E-02	1.7199E-01	-5.0913E-05	-1.9222E-06	4.8591E-02
2.6000E-02	1.2812E-01	9.9073E-05	-1.8945E-06	4.8628E-02
2.7000E-02	8.6381E-02	2.0605E-04	-1.7384E-06	4.8661E-02
2.8000E-02	4.8903E-02	2.7326E-04	-1.4956E-06	4.8688E-02
2.9000E-02	1.7083E-02	3.0575E-04	-1.2035E-06	4.8706E-02
3.0000E-02	-8.3505E-03	3.0957E-04	-8.9370E-07	4.8717E-02
3.1000E-02	-2.7237E-02	2.9123E-04	-5.9173E-07	4.8723E-02
3.2000E-02	-3.9869E-02	2.5718E-04	-3.1647E-07	4.8725E-02
3.3000E-02	-4.6872E-02	2.1337E-04	-8.0607E-08	4.8725E-02
3.4000E-02	-4.9094E-02	1.6503E-04	1.0878E-07	4.8725E-02
3.5000E-02	-4.7496E-02	1.1646E-04	2.4939E-07	4.8726E-02
3.6000E-02	-4.3070E-02	7.0981E-05	3.4274E-07	4.8727E-02
3.7000E-02	-3.6762E-02	3.0946E-05	3.9318E-07	4.8728E-02
3.8000E-02	-2.9426E-02	-2.2014E-06	4.0694E-07	4.8730E-02
3.9000E-02	-2.1787E-02	-2.7807E-05	3.9130E-07	4.8731E-02
4.0000E-02	-1.4424E-02	-4.5870E-05	3.5385E-07	4.8733E-02
4.1000E-02	-7.7627E-03	-5.6890E-05	3.0191E-07	4.8734E-02
4.2000E-02	-2.0877E-03	-6.1725E-05	2.4213E-07	4.8734E-02
4.3000E-02	2.4484E-03	-6.1447E-05	1.8017E-07	4.8735E-02
4.4000E-02	5.8030E-03	-5.7223E-05	1.2055E-07	4.8735E-02
4.5000E-02	8.0213E-03	-5.0220E-05	6.6645E-08	4.8735E-02
4.6000E-02	9.2138E-03	-4.1523E-05	2.0674E-08	4.8735E-02
4.7000E-02	9.5335E-03	-3.2084E-05	-1.6155E-08	4.8735E-02
4.8000E-02	9.1565E-03	-2.2688E-05	-4.3510E-08	4.8735E-02
4.9000E-02	8.2650E-03	-1.3942E-05	-6.1751E-08	4.8735E-02
5.0000E-02	7.0338E-03	-6.2714E-06	-7.1755E-08	4.8735E-02
5.1000E-02	5.6207E-03	6.5041E-08	-7.4740E-08	4.8735E-02
5.2000E-02	4.1595E-03	4.9543E-06	-7.2108E-08	4.8735E-02
5.3000E-02	2.7571E-03	8.4040E-06	-6.5312E-08	4.8735E-02
5.4000E-02	1.4923E-03	1.0515E-05	-5.5747E-08	4.8736E-02
5.5000E-02	4.1684E-04	1.1452E-05	-4.4674E-08	4.8736E-02

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13. ABSTRACT

A study of a multimode third order instrument servomechanism is presented. Particular emphasis is placed on the effects of the initial condition of a compensating network, switched in as the system enters a linear operating mode, upon the behaviour of the output. The optimum values for the initial condition as a function of velocity and acceleration of the servomotor is derived, using the integral squared error as a criteria. The results are confirmed with a computer simulation.

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